Printed Pages : 4

NAS-103

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 199125

Roll No. | | | | | | | | | | |

B. Tech.

(SEM. I) (ODD SEM.) THEORY

EXAMINATION, 2014-15

ENGG. MATHEMATICS-I

Time : **3** Hours] [Total Marks : **100**

1 Attempt any **FOUR** parts : **5x4=20**

a) If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$ prove that

$$(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0.$$

b) Prove that $xu_x + yu_y = \frac{5}{2}\tan u$ if

$$u = \sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right).$$

c) If $V=f(2x-3y, 3y-4z, 4z-2x)$ prove that

$$6V_x + 4V_y + 3V_z = 0.$$

d) Find $\frac{du}{dt}$ as a total derivative and verify the result by direct substitution if $u = x^2+y^2+z^2$ and $x = e^{2t}$,

$$y = e^{2t}\cos 3t,\ z = e^{2t}\sin 3t.$$

e) Trace the curve $y^2(2a-x) = x^3$.

f) Find the curve $r^2 = a^2\cos 2\theta$.

2 Attempt any **TWO** parts : **10x2=20**

a) Expand $e^x\log(1+y)$ in powers of x and y upto terms of third degree.

b) A rectangle box open at the top is to have 32cubic ft. Find the dimensions of the box requiring least material for its construction.

c) Find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ if $x=\sqrt{vw}, y=\sqrt{uw}, z=\sqrt{uv}$ and

$$u = r\sin\theta\cos\phi, v = r\sin\theta\sin\phi, w = r\cos\theta.$$

3 Attempt any **TWO** parts : **10x2=20**

a) Reduce A to Echelon form and then to its row canonical form where $A=\begin{pmatrix}1 & 3 & -1 & 2\\ 0 & 11 & -5 & 3\\ 2 & -5 & 3 & 1\\ 4 & 1 & 1 & 5\end{pmatrix}$. Hence find the rank of A.

b) Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{pmatrix}$. Hence find A^{-1}.

c) Solve by calculating the inverse by elementary row operations : $x_1+x_2+x_3+x_4=0$, $x_1+x_2+x_3-x_4=4$, $x_1+x_2-x_3+x_4=-4$, $x_1-x_2+x_3+x_4=2$.

4 Attempt any **TWO** parts : **10x2=20**

a) Determine the area bounded by the curves xy = 2, $4y=x^2$ and y = 4.

b) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

c) Find the volume and the mass contained in the solid region in the first octant of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ if the density at any point $\rho(x,y,z)=kxyz$.

5 Attempt any **TWO** parts : **10x2=20**

a) If $u=x+y+z$, $v=x^2+y^2+x^2$, $w=yz+zx+xy$. Prove that grad u, grad v and grad w are coplanar.

b) Verify Stokes theorem for $F=(x^2+y^2)I-2xyJ$ taken around the rectangle bounded by the lines $x=\pm a$, $y=0$, $y=b$

c) Evaluate $\int_S (yzI+zxJ+xyK)$ ds where S is the surface of the sphere $x^2+y^2+x^2=a^2$ in the first octant.
